

## **Problem of the Month: *Once Upon a Time***

The Problems of the Month (POM) are used in a variety of ways to promote problem-solving and to foster the first standard of mathematical practice from the Common Core State Standards: “Make sense of problems and persevere in solving them.” The POM may be used by a teacher to promote problem-solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem-solving as a key aspect of doing mathematics. It can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem-solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth grade students have success and understanding. Level C may stretch sixth and seventh grade students. Level D may challenge most eighth and ninth grade students, and Level E should be challenging for most high school students. These grade- level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem-solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

### **Overview:**

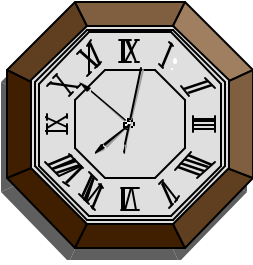
In the Problem of the Month *Once Upon a Time*, students use measurement, number properties, and circular geometry to solve problems involving time and angles. The mathematical topics that underlie this POM are time measurement, including conversion between years, months, weeks, days, hour, minutes, and seconds; modular arithmetic that involves divisibility and remainders; pattern recognition; as well as circles and angular measurement.

In level A of the POM, students are presented with the task of determining how many minutes will pass before the large hand catches up to the small hand on a clock. Their task involves counting up and understanding how a clock measures time. At level B, the students are asked to convert their age from years into seasons, months, and weeks. Students are also asked to determine what day number the current date is in the year. In level C, students are given a problem that requires an understanding of divisibility and may be determined by using knowledge of relatively prime factors. In level D, the student is presented with a problem that involves three different size alarm clocks that ring at varied intervals. The task is to determine if or when the three clocks chime simultaneously. Some students may use an understanding of modular division to solve the problem. In level E, students are asked to determine the times in a day that the clock hands form an angle of 48 degrees.

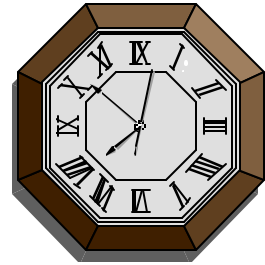
**Mathematical Concepts:**

This POM involves cyclical patterns and measurement. The most common cyclical pattern is time measurement. Time measurement and circles are intertwined. The fact that circles have 360 degrees has its roots in the original calendars that determined a year was 360 days. Cyclical relationships are often represented by circles and their component parts such as angles and sectors. Cyclical relationships are common throughout mathematics with links between number, geometry, algebra, trigonometry, and measurement.

## Problem of the Month



### Once Upon a Time



#### **Level A:**

When it is four o'clock, how many minutes must pass before the big hand (minute hand) gets to where the little hand (hour hand) was at four o'clock?

How did you figure it out?

When it is six-thirty, how many minutes must pass before the big hand (minute hand) gets to where the little hand (hour hand) was at six-thirty?

Explain the way you figured it out.

**Level B:**

How old are you?

- State your answer in years.
- State your answer in seasons.
- State your answer in months.
- State your answer in weeks.

What date is it?

What number day of the year is it?

How many more days until January 1?

### **Level C:**

I met a man who said if you can guess my age, I will pay you one dollar for each year that I have lived. I will also give you two hints. If you take my age and divide it by any odd number greater than 1 and less than 9 you will get a remainder of 1. But if you take my age and divide it by any even number greater than 1 and less than 9 you will not get a remainder of 1.

How much money could you earn?

Explain your solution and how you know it is the only correct answer.

**Level D:**

An eccentric clockmaker built three different clocks.

The first clock was a five-minute clock designed with an alarm set to sound each time the hand reached the number 2.

The second clock was a six-minute clock designed to sound each time the hand reached the number 3.

The third clock was a seven-minute clock designed to sound each time the hand reach the number 4.

The clockmaker started the clocks simultaneously on day, and each clock began to sound at its appropriate time. Was there a time when all three clocks sounded their alarms together? If so, tell when it occurred and explain why. If not, explain why not.

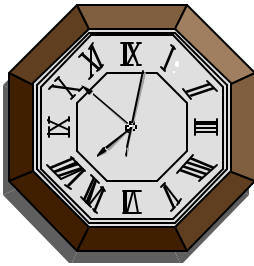
**Level E:**

The minute hand and hour hand on a clock form a  $48^\circ$  angle. What time is it?

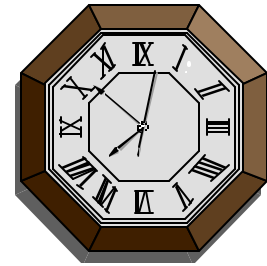
At what other times during the day do the same hands on the clock form a  $48^\circ$  angle?

How many times in a day (24 hour period) do the hands form a  $48^\circ$  angle?  
Explain your reasoning.

## Problem of the Month



# Once Upon a Time



### Primary Version Level A

**Materials:** A large demonstration clock, a small clock per group.

**Discussion on the rug:** (Teacher holds up the clock set at 4:00) "What time does this clock say?" (Teacher solicits answers from students)  
"Which is the minute hand? Which is the hour hand?" (Teacher solicits answers from students, teacher may refer to hands as big and small).  
"How do the hands move around the clock?" (Students demonstrate).  
"Which hand moves the fastest?"

**In small groups:** (Each group has an individual clock) The teacher states the following.

Set your clock to 4 o'clock. Which direction do the hands move? Which is the minute hand (big hand)? How many minutes must pass before the minute hand gets to where the hour hand (little hand) is now? Draw a picture or explain using words how you know.

Set your clock to 6:30. Which direction do the hands move? Which is the minute hand (big hand)? How many minutes must pass before the minute hand gets to where the hour hand (little hand) is now? Draw a picture or explain using words how you know.

(At the end of the investigation have students either discuss or dictate a response to this summary question)



<b>Problem of the Month</b>
<b><i>Once Upon a Time</i></b>
<b>Task Description – Level A</b>
This task challenges a student to determine how many minutes will pass before the large hand will catch up with the small hand on an analog clock. A student will understand how an analog clock measures time and will use counting principles to work on this task.
<b>Common Core State Standards Math - Content Standards</b>
<p><b><u>Counting and Cardinality</u></b>  <b>Know number names and the count sequence.</b>  K.CC.1 Count to 100 by ones and by tens.</p> <p>K.CC.5 Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.</p> <p><b><u>Operations and Algebraic Thinking</u></b>  <b>Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.</b>  K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.</p> <p><b>Add and subtract within 20.</b>  1.OA.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).</p> <p><b>Represent and solve problems involving addition and subtraction.</b>  2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.</p> <p><b><u>Measurement and Data</u></b>  <b>Tell and write time.</b>  1.MD.3 Tell and write time in hours and half-hours using analog and digital clocks.</p> <p><b>Work with time and money.</b>  2.MD.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.</p>
<b>Common Core State Standards Math – Standards of Mathematical Practice</b>
<p><b>MP.7 Look for and make use of structure.</b>  Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see <math>7 \times 8</math> equals the well-remembered <math>7 \times 5 + 7 \times 3</math>, in preparation for learning about the distributive property. In the expression <math>x^2 + 9x + 14</math>, older students can see the 14 as <math>2 \times 7</math> and the 9 as <math>2 + 7</math>. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see <math>5 - 3(x - y)^2</math> as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers <math>x</math> and <math>y</math>.</p>

**MP.8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x-1)(x+1)$ ,  $(x-1)(x^2+x+1)$ , and  $(x-1)(x^3+x^2+x+1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Problem of the Month
<b><i>Once Upon a Time</i></b>
Task Description – Level B
This task challenges a student to convert their age from years into seasons, months, and weeks. A student is also asked to determine what day number the current date is in the year. A student may use unit conversions and variables.
Common Core State Standards Math - Content Standards
<p><b><u>Operations and Algebraic Thinking</u></b></p> <p><b>Represent and solve problems involving addition and subtraction.</b></p> <p>2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.</p> <p><b><u>Number and Operations in Base Ten</u></b></p> <p><b>Use place value understanding and properties of operations to add and subtract.</b></p> <p>2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.</p> <p><b>Use place value understanding and properties of operations to perform multi-digit arithmetic.</b></p> <p>3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.</p> <p><b>Use place value understanding and properties of operations to perform multi-digit arithmetic.</b></p> <p>4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.</p> <p><b><u>Measurement and Data</u></b></p> <p><b>Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.</b></p> <p>4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. ...</p> <p>4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. ...</p> <p><b><u>Expressions and Equations</u></b></p> <p><b>Apply and extend previous understandings of arithmetic to algebraic expressions.</b></p> <p>6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.</p>
Common Core State Standards Math – Standards of Mathematical Practice
<p><b>MP.7 Look for and make use of structure.</b></p> <p>Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see <math>7 \times 8</math> equals the well-remembered <math>7 \times 5 + 7 \times 3</math>, in preparation for learning about the distributive property. In the expression <math>x^2 + 9x + 14</math>, older students can see the 14 as <math>2 \times 7</math> and the 9 as <math>2 + 7</math>. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see <math>5 - 3(x - y)^2</math> as 5 minus a</p>

positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

**MP.8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through  $(1, 2)$  with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x-1)(x+1)$ ,  $(x-1)(x^2+x+1)$ , and  $(x-1)(x^3+x^2+x+1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Problem of the Month
<b><i>Once Upon a Time</i></b>
Task Description – Level C
This task challenges a student to determine a man’s age and to explain the solution and how it is the only correct solution. Two hints are given. A student will use basic operations and understand remainders in a real-life context. A student will use an understanding of divisibility and the knowledge of relatively prime factors to work on this task.
Common Core State Standards Math - Content Standards
<p><b>Operations and Algebraic Thinking 4.OA</b>  <b>Gain familiarity with factors and multiples.</b>  4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.</p> <p><b>Analyze patterns and relationships.</b>  5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.</p> <p><b><u>The Number System</u></b>  <b>Compute fluently with multi-digit numbers and find common factors and multiples.</b>  6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.</p> <p>6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. ...</p>
Common Core State Standards Math – Standards of Mathematical Practice
<p><b>MP.7 Look for and make use of structure.</b>  Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see <math>7 \times 8</math> equals the well-remembered <math>7 \times 5 + 7 \times 3</math>, in preparation for learning about the distributive property. In the expression <math>x^2 + 9x + 14</math>, older students can see the 14 as <math>2 \times 7</math> and the 9 as <math>2 + 7</math>. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see <math>5 - 3(x - y)^2</math> as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers <math>x</math> and <math>y</math>.</p> <p><b>MP.8 Look for and express regularity in repeated reasoning.</b>  Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation <math>(y - 2)/(x - 1) = 3</math>. Noticing the regularity in the way terms cancel when expanding <math>(x-1)(x+1)</math>, <math>(x-1)(x^2+x+1)</math>, and <math>(x-1)(x^3+x^2+x+1)</math> might lead them to the general formula for the sum of a geometric</p>

series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Problem of the Month:
<b><i>Once Upon a Time</i></b>
Task Description – Level D
This task challenges a student to determine when the alarms on three different modular clocks that ring at varied intervals will all sound at the same time. A student will need a deep understanding of place value and quantities for working on this task. Some students may use an understanding of modular division to solve the problem.
Common Core State Standards Math - Content Standards
<p><b><u>Operations and Algebraic Thinking</u></b>  <b>Gain familiarity with factors and multiples.</b>  4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.</p> <p><b>Analyze patterns and relationships.</b>  5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.</p> <p><b><u>The Number System</u></b>  <b>Compute fluently with multi-digit numbers and find common factors and multiples.</b>  6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.   6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 12. ...</p> <p><b><u>Expressions and Equations</u></b>  <b>Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</b>  7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.   <b>Analyze and solve linear equations and pairs of simultaneous linear equations.</b>  8.EE.7 Solve linear equations in one variable.</p>
Common Core State Standards Math – Standards of Mathematical Practice
<p><b>MP.7 Look for and make use of structure.</b>  Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see <math>7 \times 8</math> equals the well-remembered <math>7 \times 5 + 7 \times 3</math>, in preparation for learning about the distributive property. In the expression <math>x^2 + 9x + 14</math>, older students can see the 14 as <math>2 \times 7</math> and the 9 as <math>2 + 7</math>. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see <math>5 - 3(x - y)^2</math> as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers <math>x</math> and <math>y</math>.</p> <p><b>MP.8 Look for and express regularity in repeated reasoning.</b>  Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they</p>

are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through  $(1, 2)$  with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x-1)(x+1)$ ,  $(x-1)(x^2+x+1)$ , and  $(x-1)(x^3+x^2+x+1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.



Problem of the Month
<b><i>Once Upon a Time</i></b>
Task Description – Level E
This task challenges a student to determine the times in a day when the hands of an analog clock will form a 48 degree angle. A student will use their understanding of angle and angular measures to work on this task.
Common Core State Standards Math - Content Standards
<p><b><u>Geometry</u></b>  <b>Draw and identify lines and angles, ...</b>  4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), ...</p> <p><b>Ratios and Proportional Relationships 6.RP</b>  <b>Understand ratio concepts and use ratio reasoning to solve problems.</b>  6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”</p> <p>6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.  d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.</p> <p><b><u>The Number System</u></b>  <b>Compute fluently with multi-digit numbers and find common factors and multiples.</b>  6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.</p> <p>6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 12. ...</p> <p><b><u>Expressions and Equations</u></b>  <b>Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</b>  7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</p> <p><b>Analyze and solve linear equations and pairs of simultaneous linear equations.</b>  8.EE.7 Solve linear equations in one variable.</p> <p><b><u>High School – Algebra - Creating Equations</u></b>  <b>Create equations that describe numbers or relationships.</b>  A-CED.1 Create equations and inequalities in one variable and use them to solve problems.</p> <p><b><u>High School – Geometry - Congruence</u></b>  <b>Experiment with transformations in the plane</b>  G-CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.</p> <p>G-CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.</p> <p><b>Make geometric constructions</b></p>

G-CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

### Common Core State Standards Math – Standards of Mathematical Practice

#### **MP.7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well-remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

#### **MP.8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through  $(1, 2)$  with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x-1)(x+1)$ ,  $(x-1)(x^2+x+1)$ , and  $(x-1)(x^3+x^2+x+1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Problem of the Month
<b><i>Once Upon a Time</i></b>
Task Description – Primary Level
<p>This task challenges a student to work with analog clocks to lay a foundation for understanding time. A discussion is held so that a student would understand the measurement of time with an analog clock. The teacher asks the class, when shown an analog clock, which is the minute hand and which is the hour hand. A teacher also needs to ask about the movement of the hands and which hand moves faster than the other. Each group of students has their own individual clock which they set at 4 o'clock and are asked how many minutes must pass before the minute hand gets to where the hour hand is now. Each student is asked to draw a picture or explain using words how they know. The groups are then instructed to set the clock at 6:30 with affirmation of the movement of the hands and question of "How many minutes must pass before the minute hand gets to where the hour hand is now?" Each student draws a picture or uses words to explain how they know.</p>
Common Core State Standards Math - Content Standards
<p><b><u>Counting and Cardinality</u></b>  <b>Know number names and the count sequence.</b>  K.CC.1 Count to 100 by ones and by tens.  K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.</p> <p><b><u>Operations and Algebraic Thinking</u></b>  <b>Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.</b>  K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.</p> <p><b>Add and subtract within 20.</b>  1.OA.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).</p> <p><b>Represent and solve problems involving addition and subtraction.</b>  2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.</p> <p><b><u>Measurement and Data</u></b>  <b>Tell and write time.</b>  1.MD.3 Tell and write time in hours and half-hours using analog and digital clocks.</p> <p><b>Work with time and money.</b>  2.MD.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.</p>
Common Core State Standards Math – Standards of Mathematical Practice
<p><b>MP.7 Look for and make use of structure.</b>  Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see <math>7 \times 8</math> equals the well-remembered <math>7 \times 5 + 7 \times 3</math>, in preparation for learning about the distributive property. In the expression <math>x^2 + 9x + 14</math>, older students can see the 14 as <math>2 \times 7</math> and the 9 as <math>2 + 7</math>. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview</p>

and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(\mathbf{x} - \mathbf{y})^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $\mathbf{x}$  and  $\mathbf{y}$ .

**MP.8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x-1)(x+1)$ ,  $(x-1)(x^2+x+1)$ , and  $(x-1)(x^3+x^2+x+1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.